

2.6 Trig and limits

2014-09-26 day 23

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \cdot \frac{1 + \cos x}{1 + \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x(1 + \cos x)}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1 + \cos x}$$

Red annotations:
 - An arrow points from the $\sin x$ in the numerator to the $\sin x$ in the denominator of the second fraction.
 - An arrow points from the x in the denominator of the first fraction to the $\sin x$ in the numerator of the first fraction, with a red '1' below it.
 - An arrow points from the $1 + \cos x$ in the denominator of the second fraction to the $1 + 1 = 2$ below it.

Big
Trig
Equality
 $\sin^2 x + \cos^2 x = 1$

$$= \frac{0}{2} = 0$$

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$$\begin{aligned}
 27) \quad & \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \lim_{x \rightarrow 0} \frac{x^2(1 + \cos x)}{1 - \cos^2 x} \\
 & = \lim_{x \rightarrow 0} \frac{x^2(1 + \cos x)}{\sin^2 x} = \lim_{x \rightarrow 0} \underbrace{\frac{x}{\sin x}}_1 \cdot \underbrace{\frac{x}{\sin x}}_1 \cdot \underbrace{\frac{1 + \cos x}{1}}_{\frac{1+1}{1}=2} \\
 & \quad = 2
 \end{aligned}$$

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$$\text{20) } \lim_{x \rightarrow 0} \frac{\sin^2 x}{3x^2} = \lim_{x \rightarrow 0} \underbrace{\frac{1}{3}}_{\frac{1}{3}} \cdot \underbrace{\frac{\sin x}{x}}_1 \cdot \underbrace{\frac{\sin x}{x}}_1 = \frac{1}{3}$$

$$\text{19) } \lim_{x \rightarrow 0^-} \frac{\sin x}{|x|} = \lim_{x \rightarrow 0^-} \frac{\sin x}{-x} = -1$$

$$|x| = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$$

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$$24) \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1} = 0$$

$\swarrow \quad \searrow$
 $1 \quad 0$

$$25) \lim_{x \rightarrow 0} \frac{x}{\tan x} = \lim_{x \rightarrow 0} \frac{x}{\frac{\sin x}{\cos x}} = \lim_{x \rightarrow 0} \frac{x \cos x}{\sin x}$$
$$= \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \frac{\cos x}{1} = 1$$

$\swarrow \quad \searrow$
 $1 \quad 1$

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$$33) \lim_{x \rightarrow 0^+} \cos\left(\frac{1}{x}\right) = \cos\left(\lim_{x \rightarrow 0^+} \frac{1}{x}\right)$$

$$= \cos(\text{infinity-ish}) = \text{DNE}$$

(because of
oscillation)

